Section 6.4 Solutions

2. work, $w = [(200\ kg)(9.8\ m/s^2)](3\ m) = \boxed{5{,}880\ J}$
$= [\text{Force}] \times (\text{Distance})$

6. If we use $n=4$, then we have the following midpoints and the corresponding values of the function f

$\bar{x}_i$	6	10	14	18
$f(\bar{x}_i)$	5.8	8.8	8.2	5.2

with $\Delta x = \frac{20-4}{4} = 4$

Then work $W = \int_4^{20} f(x)\,dx \approx \Delta x\left[f(6)+f(10)+f(14)+f(18)\right]$
$= 4[5.8+8.8+8.2+5.2] = \boxed{112\ J}$

12. This problem is mostly about algebra. We have two equations in two unknowns. This is how I did it.

Given: $\int_a^b kx\,dx = 6$ and $\int_b^c kx\,dx = 10$, where

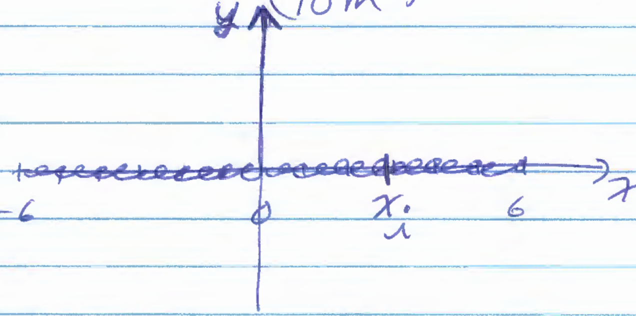
$a = 0.1 -$ Rest length, in meters
$b = a + 0.02$, in meters
$c = a + 0.04$, in meters

$\int_a^b kx\,dx = \frac{1}{2}k(b^2-a^2) = \frac{1}{2}k(b+a)(b-a) = \frac{1}{2}k(2a+0.02)(0.02)$
$\int_b^c kx\,dx = \frac{1}{2}k(c^2-b^2) = \frac{1}{2}k(c+b)(c-b) = \frac{1}{2}k(2a+0.06)(0.02)$

Now we solve the equations $\frac{1}{2}k(2a+0.02)(0.02) = 6$ and $\frac{1}{2}k(2a+0.06)(0.02) = 10$ simultaneously to find $\boxed{k = 10{,}000}$, $a = 0.02\ m$, so Rest length $=$
$0.10 - a = \boxed{0.08\ m}$ or 8 cm

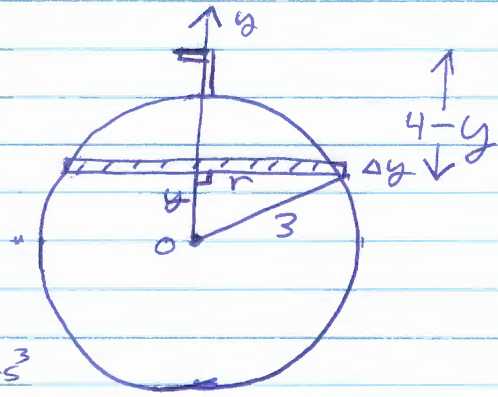
14. The (linear) weight density of chain $= \left(\frac{80\,kg}{10\,m}\right)(9.8\,m/s^2)$

$= 78.4\ N/m$



Using the coordinate system shown to the right, only the chain to the right of 0 will be raised. In fact, the chain at x_i will be raised exactly x_i meters (hence my choice of coordinates). Then the work done raising half the chain is

$$w = \int_0^6 78.4x\,dx = 39.2x^2\Big|_0^6 = \boxed{1{,}411.2\ J}$$



22. The slices of the sphere shown in the picture are disks of radius r, where $r^2 = 3^2 - y^2$ using the Pythagorean Thm.

Now, ΔV_i = Volume of i^{th} disk
$= \pi r_i^2 \Delta y = \pi(9 - y_i^2)\Delta y$ meters^3

ΔF_i = weight of i^{th} disk
$= (1000\ kg/m^3)(9.8\ m/s^2)\Delta V_i$ Newtons

ΔW_i = Work done on i^{th} disk
$= (\Delta F_i)(4 - y)$

Total work, $W = \int_{-3}^{3} (1000)(9.8)\pi(9-y^2)(4-y)\,dy$

$= 9800\pi \int_{-3}^{3} (36 - 9y - 4y^2 + y^3)\,dy$

$= 19600\pi \int_0^3 (36 - 4y^2)\,dy$ (Why?)

$= \boxed{1{,}411{,}200\pi\ J}$